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Software Section

BARON: A General Purpose Global Optimization Software Package

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Abstract. The Branch-And-Reduce Optimization Navigator (BARON) is a computational system for facilitating the solution of nonconvex optimization problems to global optimality. We provide a brief description of the algorithms used by the software, describe the types of problems that can be currently solved and summarize our recent computational experience. BARON is available by anonymous ftp from `aristotle.me.uiuc.edu`.

Key words: Branch-and-bound, polynomial programming, multiplicative programming, mixed-integer nonlinear programming, quadratic programming, fixed-charge problem.

1. Introduction

The analysis and design of physical and engineering systems frequently require the optimal solution of nonconvex models. This report summarizes our efforts aimed at the development of an easily expandable, general purpose, high performance global optimization software to support research in the development of global optimization algorithms and their applications.

2. Basic Algorithms

We consider general nonlinear and mixed-integer nonlinear programs:

$$
\begin{aligned}
(P): \qquad & \text{glob min} \quad f(x) \\
& \text{s.t.} \quad g(x) \leq 0 \\
& \qquad\quad x \in X
\end{aligned}
$$

where $f: X \rightarrow \Re$, $g: X \rightarrow \Re^m$, and $X \subset \Re^n$.

The main algorithmic assumption is that there exists a relaxation, R, of P:

$$
\begin{aligned}
(R): \qquad & \min \quad \bar{f}(x) \\
& \text{s.t.} \quad \bar{g}(x) \leq 0 \\
& \qquad\quad x \in \bar{X}
\end{aligned}
$$

where $\bar{f} : \bar{X} \rightarrow \Re$, $\bar{g} : \bar{X} \rightarrow \Re^{\bar{m}}$, $X \subseteq \bar{X} \subset \Re^{\bar{n}}$, such that, for any x feasible to P, $\bar{f}(x) \leq f(x)$, and $\{x : \; g(x) \leq 0, \; x \in X\} \subseteq \{x : \bar{g}(x) \leq 0, \; x \in \bar{X}\}$. It is further assumed that a conventional optimization algorithm is available for solving R.

For given P and R, BARON navigates its way through problem specific subroutines. Its global optimization strategy integrates conventional branch-and-bound with a wide variety of range reduction tests. These tests are applied to every subproblem of the search tree in pre- and post-processing steps to contract the search space and reduce the relaxation gap. Many of the reduction tests are based on duality and are applied when R is convex and solved by an algorithm that provides the dual, in addition to the primal, solution of R. Another crucial component of the software is the implementation of heuristic techniques for the approximate solution of optimization problems that yield improved bounds for the problem variables. Finally, the algorithm incorporates a number of compound branching schemes that accelerate convergence of standard branching strategies.

Detailed descriptions of the core component as well as the specialized modules are provided in [7, 6, 8, 4]. In [6] the algorithm is shown to terminate finitely with an approximate solution that is within ϵ tolerance from the global optimal solution $(\epsilon > 0)$. *Finite* termination with the *exact* global optimum $(\epsilon = 0)$, is also proven for linearly constrained problems with concave objective functions [8].

3. Areas Covered by the Software

3.1. Core Component

To maximize the system flexibility and expandability, we have developed the code in a way that problem specific subroutines are supplied to the core of the software. Any global optimization problem can therefore be solved with BARON so long as lower and upper bounding subroutines are provided by the user. In this way, the core system is capable of solving very general problems although this requires some coding by the user.

3.2. Specialized Modules

In addition to the general purpose core, ready to use specialized modules have been developed for several important problem classes that are described in this section. These modules utilize the generic capabilities of the core component, require no coding from the user and accept problem data in a simple input form as described in the documentation of BARON.

3.2.1. *Univariate Polynomial Programming*

$$
\begin{aligned}
(POLY): \quad & \text{glob min} \;\; f(x) = \sum_{i=0}^{k} c_i x^i \\
& \text{s.t.} \quad l \le x \le u
\end{aligned}
$$

where $x \in \Re$, $c \in \Re^k$, $l \in \Re$, $u \in \Re$.

3.2.2. *Linear Multiplicative Programming*

$$
\begin{aligned}
(LMP): \quad & \text{glob min} \;\; f(x) = \prod_{i=1}^{p} f_i(x) = \prod_{i=1}^{p} \left(c_i^t x + c_{i0}\right) \\
& \text{s.t.} \quad a \le Ax \le b \\
& \qquad\quad c_i^t x + c_{i0} \ge 0 \quad (i = 1, \ldots, p)
\end{aligned}
$$

where $x \in \Re^n$, $c_i \in \Re^n$ and $c_{i0} \in \Re$ $(i = 1, \ldots, p)$, $a \in \Re^m$, $b \in \Re^m$, and $A \in \Re^{m \times n}$.

3.2.3. *Separable Concave Quadratic Programming*

$$
\begin{aligned}
(SCQP): \quad & \text{glob min} \;\; f(x) = \sum_{i=1}^{n} (c_i x_i + q_i x_i^2) \\
& \text{s.t.} \quad a \le Ax \le b \\
& \qquad\quad l \le x \le u
\end{aligned}
$$

where $x \in \Re^n$, $c \in \Re^n$, $q \in \Re^n_-$, $a \in \Re^m$, $b \in \Re^m$, $l \in \Re^n$, $u \in \Re^n$ and $A \in \Re^{m \times n}$.

3.2.4. *Fixed-Charge Programming*

$$
\begin{aligned}
(FCP): \quad & \text{glob min} \;\; f(x) = \sum_{i=1}^{n} f_i(x_i) \\
& \text{s.t.} \quad a \le Ax \le b \\
& \qquad\quad l \le x \le u
\end{aligned}
$$

where $x \in \Re^n$, $f_i(x_i)$ equals $c_i + q_i x_i$ if $x_i > 0$ and 0 otherwise, $c \in \Re^n_+$, $q \in \Re^n$, $a \in \Re^m$, $b \in \Re^m$, $l \in \Re^n_+$, $u \in \Re^n_+$ and $A \in \Re^{m \times n}$.

3.2.5. *Problems with Power Economies of Scale*

$$
\begin{aligned}
(PES): \quad & \text{glob min} \; f(x) = \sum_{i=1}^{n} c_i x_i^{q_i} \\
& \text{s.t.} \quad a \le Ax \le b \\
& \qquad\quad l \le x \le u
\end{aligned}
$$

where $x \in \Re^n$, $c \in \Re^n_+$, $q \in \Re^n$, $0 < q \le 1$, $a \in \Re^m$, $b \in \Re^m$, $l \in \Re^n_+$, $u \in \Re^n_+$ and $A \in \Re^{m\times n}$.

4. Computational Experience

Extensive computational results with the algorithm are presented in [7, 6, 8] for standard global optimization test problems, univariate polynomial programs, linear multiplicative programs, mixed-integer nonlinear programs and concave quadratic programs. Applications have included chemical process design problems [7], the long range planning of chemical process networks [4], the chip layout and compaction problem [1], and the design of just-in-time manufacturing systems [2].

The computational results indicate that certain global optimization problems, which until recently were thought to require advanced (such as parallel or distributed) computers for their solution, can now be solved on a standard engineering workstation using BARON. The specialized modules solve problems with hundreds of variables in a matter of a few minutes on an IBM RS/6000 66MHz-Power PC. Table I presents the sizes and solution times for some large-scale problems that have been solved with $\epsilon = 10^{-6}$.

Table I. Computational experience with specialized modules.

Module	p	m	n	CPU min	Reference
LMP	5	200	200	4	[6]
SCQP		50	500	6	[8]
FCP		417	745	7	[4, 8]
PES		331	505	5	[4]

5. Hardware and Software Requirements

We currently provide codes for IBM RS/6000, SUN, SGI and DEC workstations under the AIX and UNIX operating systems. BARON is extremely portable and we can provide versions for additional platforms upon request. The specialized modules use OSL [3] and MINOS 5.4 [5] for solving linear and nonlinear programming subproblems, respectively.

6. Access to BARON

The program and several test problems have been available by anonymous ftp from `aristotle.me.uiuc.edu` since March 2, 1995. Access statistics to the site for the first 7 months are summarized in Table II. The software can be used and distributed freely as long as it is not sold for profit or incorporated in commercial products. Comments and suggestions are welcome by the author.

Table II. Internet access to BARON.

	Mar	Apr	May	Jun	Jul	Aug	Sep	Total
EDU	130	79	79	110	75	88	101	662
COM	20	0	2	13	24	12	19	90
Others	18	19	22	28	52	64	42	245
USA	59	29	46	78	58	43	69	382
Europe	46	32	36	37	78	76	81	386
Others	63	37	21	36	15	45	12	229
Total	168	98	103	151	151	164	162	997

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